Process Modeling of Composites by Resin Transfer Molding: Sensitivity Analysis for Isothermal Considerations

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ARL-TR-2686   March 2002

20020402 174

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Abstract

The resin transfer molding (RTM) manufacturing process consists of either of two considerations; the first is the fluid flow analysis through a porous fiber preform where the location of the flow front is of fundamental importance, and the second is combined flow/heat transfer analysis. For preliminary design purposes and the case of relatively large molds, isothermal considerations seem fairly representative of the physical situation. The continuous sensitivity formulations are developed for the process modeling of composites manufactured by RTM to predict, analyze, and the optimize the manufacturing process. Attention is focused here on developments for isothermal flow simulations, and illustrative examples are presented for sensitivity analysis applications which help serve as a design tool in the process modeling stages.

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1. Introduction

In manufacturing processes, there are many factors which affect the resulting product. Many times, the job of an engineer is to find the optimum process by weighing the various input factors and their affect on the outcome. In the recent past, the optimization method has changed from heuristic trial and error to more rigorous computational methods employing the finite element method. Trial and error is still used with finite elements, but only the computer time is involved and costly trial runs can be eliminated. A more stringent method for optimizing manufacturing processes which does not require the level of intuition as trial and error is via sensitivity analysis. With properly designed software, the engineer can define the limits of input parameters; through iteration and optimization, an optimal solution is computed.

Much effort has gone into the numerical approaches for the resin transfer molding (RTM) process. Some of the methods include the marker and cell (MAC) approach [1, 2] which has been employed in metal casting simulations, the volume of fluid (VOF) approach [3] also commonly used in metal casting simulations, the control volume-finite element (CV-FE) explicit approach, and the pure finite element (Pure FE) implicit approach by Mohan et al. [4–8]. The pure finite element approach has proven to be of practical importance and with improved physical attributes when compared to the traditional approaches. On the other hand, little effort has gone into developing sensitivity analysis for RTM process flow modeling. Some preliminary work utilizing the implicit finite element method [9] and LIMS [10] for sensitivity analysis appears in the literature. These articles show the preliminary developments of the sensitivity equations for isothermal flow modeling with the continuous sensitivity equation (CSE) and examine the results obtained from the numerical analysis.

In this report, the CSE is formulated for RTM process modeling of composites. Attention is focused here on isothermal situations, which is an acceptable practice for significantly large

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molds and helps serve a useful purpose for preliminary design stages. Essentially, the physical problem is that of the resin flow through a porous fiber network and the accurate tracking of the moving fluid flow front. The CSE approach is useful for design as it starts from the original governing model equation and with the finite element method of discretization, the system of finite element sensitivity equations can be formulated and solved. It happens that this system of sensitivity equations is always linear; also, they can be solved as a post-processing phase so that the computational requirements and code modifications are minimal. After the CSE is formulated for the isothermal RTM filling model, the results are used to analyze and verify a sample problem. The sensitivity results are also used to compute the value of a variable material property, such as permeability or viscosity.

2.   Isothermal Resin Transfer Molding

Under certain circumstances, the assumption of isothermal flow is reasonable for use in resin transfer molding. In this report the investigated sensitivities in the isothermal RTM filling section are pressure sensitivity, $S_P \equiv \frac{\partial P}{\partial p}$, where p is the sensitivity parameter, with respect to inlet flow rate q, permeability $\bar{K}$, and viscosity μ. Also, the fill time sensitivity, $S_{tfill} \equiv \frac{\partial t_{fill}}{\partial p}$, with respect to inlet pressure, permeability, viscosity, and inlet location are investigated. The governing model equations for isothermal RTM process modeling are briefly described next. The continuity equation is given as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \tag{1}$$

The Gauss theorem is used to convert the volume integral to a surface integral so the continuity equation is given in integral form as

$$\int_{\Omega} \frac{\partial \rho}{\partial t} \, d\Omega + \int_{\Gamma} \rho (\mathbf{u} \cdot \mathbf{n}) \, d\Gamma = 0 \tag{2}$$

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From practical considerations and with improved physical attributes, it is possible to track the flow front following an implicit pure finite element mold filling formulation as described by Mohan et al. [6]. Here the variable Ψ, namely the fill factor, is introduced as

$$\int_\Omega \frac{\partial}{\partial t}(\rho\Psi)\,d\Omega + \int_\Gamma \rho\Psi(\mathbf{u}\cdot\mathbf{n})\,d\Gamma = 0 \tag{3}$$

Using the product rule on the first term of equation (3) and the fact that for an incompressible flow, $\frac{\partial\rho}{\partial t} = 0$, it yields

$$\int_\Omega \rho\frac{\partial\Psi}{\partial t}\,d\Omega + \int_\Gamma \rho\Psi(\mathbf{u}\cdot\mathbf{n})\,d\Gamma = 0 \tag{4}$$

Equivalently, for an incompressible fluid the result is given as

$$\int_\Omega \frac{\partial\Psi}{\partial t}\,d\Omega + \int_\Gamma \Psi(\mathbf{u}\cdot\mathbf{n})\,d\Gamma = 0 \tag{5}$$

Applying the Gauss theorem to the second term of equation (5), yields the representation

$$\int_\Omega \frac{\partial\Psi}{\partial t}\,d\Omega + \int_\Omega \Psi\nabla\cdot\mathbf{u}\,d\Omega = 0 \tag{6}$$

Moving the velocity term to the right hand side yields

$$\int_\Omega \frac{\partial\Psi}{\partial t}\,d\Omega = -\int_\Omega \Psi\nabla\cdot\mathbf{u}\,d\Omega \tag{7}$$

Darcy's Law is given as

$$\mathbf{u} = -\frac{\bar{\mathbf{K}}}{\mu}\cdot\nabla P \tag{8}$$

where $\bar{\mathbf{K}}$ is the permeability tensor of the fiber preform which is defined appropriately for two and three-dimensional preform considerations. Upon substituting equation (8) into

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equation (7), the transient filling model equation is given as

$$\int_{\Omega} \frac{\partial \Psi}{\partial t}\, d\Omega = \int_{\Omega} \Psi \nabla \cdot \left(\frac{\bar{\mathbf{K}}}{\mu} \cdot \nabla P \right) d\Omega \qquad (9)$$

It should be noted here that the viscosity is not a function of temperature, and hence the isothermal limitations of the model. The boundary conditions for equation (9) are

$$\frac{\partial P}{\partial n} = 0 \text{ on mold walls}$$

$$P = 0 \text{ at flow front}$$

$$P = P_0 \text{ prescribed pressure at inlet} \qquad (10)$$

$$\text{or}$$

$$q = q_0 \text{ prescribed flow rate at inlet}$$

where P_0 and q_0 represent pressure and flow rate at the inlet(s), respectively. The two initial conditions required to solve equation (9) are given as

$$\Psi = 1 \text{ at inlet}$$
$$\qquad (11)$$
$$\Psi(t = 0) = 0 \text{ elsewhere}$$

At this point it should be noted that the pressure gradient in the unfilled nodes is negligible. This implies that equation (9) is only solved for the completely filled nodes (i.e., $\Psi = 1$). Hence,

$$\int_{\Omega} \frac{\partial \Psi}{\partial t}\, d\Omega = \int_{\Omega} (1) \nabla \cdot \left(\frac{\bar{\mathbf{K}}}{\mu} \cdot \nabla P \right) d\Omega \qquad (12)$$

In order to solve the isothermal problem, the finite element method is employed as

$$\int_{\Omega} \mathbf{W}^T \frac{\partial \Psi}{\partial t}\, d\Omega = \int_{\Omega} \mathbf{W}^T \left(\nabla \cdot \frac{\bar{\mathbf{K}}}{\mu} \cdot \nabla P \right) d\Omega \qquad (13)$$

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Applying the Gauss-Green formula to equation (13) yields

$$\int_\Omega \mathbf{W^T} \frac{\partial \Psi}{\partial t} \, d\Omega + \int_\Omega \nabla \mathbf{W^T} \frac{\bar{\mathbf{K}}}{\mu} \cdot \nabla P \, d\Omega = \int_\Gamma \mathbf{W^T} \frac{\bar{\mathbf{K}}}{\mu} \cdot \nabla P \, d\Gamma \tag{14}$$

The weighting functions, $\mathbf{W}$, are chosen as the element shape functions $\mathbf{N}$. Both the pressure and fill factors are approximated as

$$\begin{aligned}
\mathbf{P} &= \sum_{i=1}^{\text{num. nodes}} N_i P_i \\
\Psi &= \sum_{i=1}^{\text{num. nodes}} N_i \Psi_i
\end{aligned} \tag{15}$$

where i represents the associated node number. After introducing equation (15), equation (14) can be represented as the following finite element semi-discretized equation system

$$\mathbf{C}\dot{\Psi} + \mathbf{KP} = \mathbf{q} \tag{16}$$

where

$$\begin{aligned}
\mathbf{C} &= \int_\Omega \mathbf{N^T N} \, d\Omega \\
\mathbf{K} &= \int_\Omega \mathbf{B^T} \frac{\bar{\mathbf{K}}}{\mu} \mathbf{B} \, d\Omega \\
\mathbf{q} &= \int_\Gamma \mathbf{N^T} \left(\frac{\bar{\mathbf{K}}}{\mu} \cdot \nabla P \cdot \mathbf{n} \right) d\Gamma \\
\dot{\Psi} &= \frac{\Psi_{\mathbf{n+1}} - \Psi_{\mathbf{n}}}{\Delta t}
\end{aligned} \tag{17}$$

Substituting the definition for $\dot{\Psi}$ from equation (17) into equation (16) yields

$$\mathbf{C}\left[\Psi_{\mathbf{n+1}} - \Psi_{\mathbf{n}}\right] + \Delta t \mathbf{KP} = \Delta t \mathbf{q} \tag{18}$$

The fill factor and pressure solutions are obtained through an iterative technique as described in Mohan et al. [6] for the implicit formulations employed here.

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3. Sensitivity Analysis of Isothermal Resin Transfer Molding Process

In this section the CSE is developed for isothermal RTM filling including sensitivity parameters of permeability, viscosity, inlet pressure, inlet flow rate, and inlet location. A cost function is then developed to compute the fill time of the mold so that the fill time sensitivity can be computed. Finally, the sensitivity results are verified with the use of a derived analytical solution and subsequently some numerical examples are presented. The sensitivity analysis for the resin transfer molding is derived by taking the partial derivative of Darcy's Law given by equation (8), which is subsequently coupled with the continuity equation, equation (1), to result in the quasi-steady state equation for mold filling. This equation is employed rather than the implicit pure finite element formulation shown earlier as it more directly relates to the sensitivity analysis. Thus,

$$\nabla \cdot \left(\frac{\bar{\mathbf{K}}}{\mu} \cdot \nabla P \right) = 0 \tag{19}$$

Note that equation (19) is a quasi-steady state representation. This implies that suitably small time steps must be used when solving the RTM filling problem. The boundary conditions for equation (19) are the same as those given in equation (10). The next step to solving for the sensitivity is to compute the CSE for isothermal RTM filling. The CSE is obtained by taking the partial derivative of equation (19) and the associated boundary conditions given by equation (10) with respect to an arbitrary sensitivity parameter p. Thus,

$$\frac{\partial}{\partial p} \left(\nabla \cdot \left(\frac{\bar{\mathbf{K}}}{\mu} \cdot \nabla P \right) \right) = 0 \tag{20}$$

After using the chain rule to obtain the derivatives of all terms, equation (20) becomes

$$\nabla \cdot \left[\frac{\partial \bar{\mathbf{K}}}{\partial p} \frac{1}{\mu} \cdot \nabla P + \bar{\mathbf{K}} \frac{\partial}{\partial p} \left(\frac{1}{\mu} \right) \cdot \nabla P + \frac{\bar{\mathbf{K}}}{\mu} \cdot \nabla S_P \right] = 0 \tag{21}$$

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The following are the associated boundary conditions for equation (21):

$$\frac{\partial}{\partial p}\left(\frac{\partial P}{\partial \mathbf{n}}\right) = 0 \text{ on Mold Walls}$$

$$\frac{\partial P}{\partial p} = 0 \text{ at flow front}$$

$$\frac{\partial P}{\partial p} = \frac{\partial P_0}{\partial p} \text{ for constant pressure at inlet} \tag{22}$$

or

$$\frac{\partial q}{\partial p} = \frac{\partial q_0}{\partial p} \text{ for constant flow rate at inlet}$$

where $\mathbf{n}$ is the normal to the progressing flow front. The boundary conditions for equation (21) can be rewritten from equation (22) using the definition of pressure sensitivity $S_P \equiv \frac{\partial P}{\partial p}$, as

$$\frac{\partial S_P}{\partial \mathbf{n}} = 0 \text{ on Mold Walls}$$

$$S_P = 0 \text{ at flow front}$$

$$S_P = \frac{\partial P_0}{\partial p} \text{ for constant pressure at inlet} \tag{23}$$

or

$$S_q = \frac{\partial q_0}{\partial p} \text{ for constant flow rate at inlet}$$

where the flow rate sensitivity is defined as $S_q \equiv \frac{\partial q}{\partial p}$. If the sensitivity parameter is the inlet location, then the boundary conditions are represented as

$$\frac{\partial S_P}{\partial \mathbf{n}} = 0 \text{ on Mold Walls}$$

$$S_P = 0 \text{ at flow front} \tag{24}$$

$$S_P = -\frac{\partial P}{\partial x} \text{ at inlet}$$

The inlet boundary condition from equation (24) is found by using the chain rule in the

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following manner:

$$\frac{\partial P(x(p); p)}{\partial p} = \frac{\partial P}{\partial p} + \frac{\partial P}{\partial x}\frac{\partial x}{\partial p} = \frac{\partial P_0}{\partial p} \tag{25}$$

Since $\frac{\partial P_0}{\partial p} = 0$ and $\frac{\partial x}{\partial p} = 1$, equation (24) is obtained. Employing the method of weighted residuals to derive the finite element equations, equation (21) leads to

$$\int_\Omega \mathbf{W}^T \nabla \cdot \left(\frac{\partial \bar{\mathbf{K}}}{\partial p} \frac{1}{\mu} \cdot \nabla P + \bar{\mathbf{K}} \frac{\partial}{\partial p}\left(\frac{1}{\mu}\right) \cdot \nabla P + \frac{\bar{\mathbf{K}}}{\mu} \cdot \nabla S_P \right) d\Omega = 0 \tag{26}$$

Applying the Gauss-Green formula to equation (26) yields

$$\int_\Omega \nabla \mathbf{W}^\mathbf{T} \frac{\partial \bar{\mathbf{K}}}{\partial p} \frac{1}{\mu} \cdot \nabla P \, d\Omega + \int_\Omega \nabla \mathbf{W}^\mathbf{T} \bar{\mathbf{K}} \frac{\partial}{\partial p}\left(\frac{1}{\mu}\right) \cdot \nabla P \, d\Omega + \int_\Omega \nabla \mathbf{W}^\mathbf{T} \frac{\bar{\mathbf{K}}}{\mu} \cdot \nabla S_P \, d\Omega$$

$$= \int_\Gamma \frac{1}{\mu} \mathbf{W}^\mathbf{T} \frac{\partial \bar{\mathbf{K}}}{\partial p} \cdot \nabla P \cdot \mathbf{n} \, d\Gamma + \int_\Gamma \mathbf{W}^\mathbf{T} \bar{\mathbf{K}} \frac{\partial}{\partial p}\left(\frac{1}{\mu}\right) \cdot \nabla P \cdot \mathbf{n} \, d\Gamma + \int_\Gamma \mathbf{W}^\mathbf{T} \frac{\bar{\mathbf{K}}}{\mu} \cdot \nabla S_P \cdot \mathbf{n} \, d\Gamma = 0 \tag{27}$$

The weighting functions $\mathbf{W}$ are chosen to be the same as the element shape functions $\mathbf{N}$, and interpolating S_P yields

$$\mathbf{S_P} = \sum_{i=1}^{\text{num. nodes}} N_i S_{P_i}$$

$$\mathbf{P} = \sum_{i=1}^{\text{num. nodes}} N_i P_i \tag{28}$$

$$\mathbf{W} = \mathbf{N}$$

The sensitivity finite element equation is given in the following semi-discretized equation representation

$$\frac{\partial \mathbf{K}}{\partial p}\mathbf{P} + \mathbf{K}\mathbf{S_P} = \mathbf{S_q} \tag{29}$$

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where $\frac{\partial \mathbf{K}}{\partial p}, \mathbf{K},$ and $\mathbf{S_q}$ are defined as

$$\frac{\partial \mathbf{K}}{\partial p} = \int\limits_\Omega \mathbf{B^T}\frac{\partial \bar{\mathbf{K}}}{\partial p}\frac{1}{\mu}\mathbf{B}\,d\Omega + \int\limits_\Omega \mathbf{B^T}\bar{\mathbf{K}}\left(-\frac{1}{\mu^2}\right)\frac{\partial \mu}{\partial p}\mathbf{B}\,d\Omega$$

$$\mathbf{K} = \int\limits_\Omega \mathbf{B^T}\frac{\bar{\mathbf{K}}}{\mu}\mathbf{B}\,d\Omega$$

$$\mathbf{S_q} = \int\limits_\Gamma \mathbf{N^T}\frac{\partial \bar{\mathbf{K}}}{\partial p}\frac{1}{\mu}\cdot\nabla P\cdot\mathbf{n}\,d\Gamma + \int\limits_\Gamma \mathbf{N^T}\bar{\mathbf{K}}\frac{\partial}{\partial p}\left(\frac{1}{\mu}\right)\cdot\nabla P\cdot\mathbf{n}\,d\Gamma + \int\limits_\Gamma \mathbf{N^T}\frac{\bar{\mathbf{K}}}{\mu}\cdot\nabla S_P\cdot\mathbf{n}\,d\Gamma$$

$$(30)$$

4. Cost Function Derivation With Fill Time Example

Cost functions are functions which describe a result derived from the computational procedure. Examples of these results could be the amount of porosity, shape of flow front, or the fill time. The last of these, namely the fill time, is chosen as an illustrative example for the isothermal RTM model simulations. The fill time is an important consideration in the RTM manufacturing process because it affects how much cure will occur before the mold is completely filled, which in turn affects the final structural properties of the part. The fill time also affects the rate at which parts are manufactured.

4.1 Objective

The objective here is to derive a cost function for the RTM filling process and utilize the CSE results to compute this cost function, namely, the fill time. This information can then be used later to optimize the computational model with respect to the sensitivity parameter, or to compute an unknown material property.

4.2 Computational Procedure

The fill time sensitivity is derived by first defining a function which includes the fill time and by taking the partial derivative with respect to the sensitivity parameter p and solving for $\frac{\partial t_{fill}}{\partial p}$, the fill time sensitivity. The information necessary to solve for the fill time sensitivity is then computed, and finally the fill time sensitivity cost function is added to the computational procedure. For resin transfer molding, the pressure sensitivity, S_P, is first solved for in equation (29) using the finite element method. In order to solve equation (29), the necessary boundary conditions must be applied. This includes the inlet conditions which may be constant pressure or constant flow rate and then the natural boundary conditions which state that resin mass can neither be created nor destroyed inside the manufactured part. The pressure sensitivity value is then used to compute the flow rate sensitivity at the mold inlet(s), with equation (29), after which the fill time sensitivity is computed. The function used to evaluate mold fill time is given in equation (31) [9] and is different to that described in Mathur et al. [10] which does not consider the present integration limits arising from the use of the quasi-steady state governing model equation.

$$V_{\Delta t} = \int_0^{\Delta t_f} q_{inlet} \, dt \tag{31}$$

where q_{inlet} is the flow rate at the mold inlet, $V_{\Delta t}$ is the volume of mold filled during the current time step, and Δt_f is the length of the current time step. The volume of the mold filled is evaluated at each time step; because equation (19) is a quasi-steady state function it is only valid for a given time step. To compute the fill time sensitivity, the partial derivative of equation (31) must be taken with respect to the sensitivity parameter p. Thus,

$$\frac{\partial V_{\Delta t}}{\partial p} = \frac{\partial}{\partial p} \int_0^{\Delta t_f} q_{inlet} \, dt \tag{32}$$

which yields

$$0 = \int_0^{\Delta t_f} S_{q_{inlet}} \, dt + \frac{\partial \Delta t_f}{\partial p} q_{inlet}(\Delta t_f) \tag{33}$$

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Solving for $\frac{\partial \Delta t_f}{\partial p}$, the fill time sensitivity for the current time step is defined as

$$\frac{\partial \Delta t_f}{\partial p} = \frac{\int_0^{\Delta t_f} S_{q_{inlet}} \, dt}{q_{inlet}(\Delta t_f)} \tag{34}$$

When summed over all the time steps, the fill time sensitivity, namely the cost function, is computed as

$$S_{t_f} = \sum_{AllTimeSteps} \frac{\int_0^{\Delta t_f} S_{q_{inlet}} \, dt}{q_{inlet}(\Delta t_f)} \tag{35}$$

These results can be used for optimization, which for the fill time is minimization. The method used to minimize the fill time can include conjugate gradient search methods, genetic algorithms, and the like. With these methods, the optimum value can be computed for a given sensitivity parameter. In the present study, the fill time optimization is not presented, but the fill time sensitivity results are instead used to compute material properties.

5. Computational Procedure for Isothermal RTM Filling Sensitivity Analysis

The sensitivity equations can be solved with minimal changes to existing RTM software. The current simulations were executed employing the general purpose code OCTOPUS (On Composite Technology of Polymeric Useful Structures). Only a few extra function calls need to be added to the existing code to compute the sensitivity results. The sensitivity equation to be solved is stated in equation (29) in conjunction with the boundary conditions given in equation (23). The 12-step solution procedure is outlined here, with the mold filling implicit pure finite element algorithm previously described in Ngo et al. [4] and Mohan et al. [5–8]:

1. Form the mass matrix, **C**, stiffness matrix, **K**, and the load vector, **q**, as defined in

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equation (17).

2. Form the sensitivity stiffness matrix, $\frac{\partial \mathbf{K}}{\partial \mathbf{p}}$, and the sensitivity load vector, $\mathbf{S_q}$, as defined in equation (30). The parameters necessary for this are computed by taking the partial derivative of permeability, $\bar{\mathbf{K}}$, and viscosity, μ, with respect to the sensitivity parameter, p.

3. Apply the natural boundary conditions. These are considered by stating that once a control volume is filled, the mass balance must be held, so $\mathbf{q} = \{0\}$ for all nodes.

4. Apply the prescribed boundary conditions to the stiffness matrix, $\mathbf{K}$, given in equation (10).

5. Compute and apply the prescribed sensitivity inlet conditions from equation (23). This includes prescribed S_P or S_q at the mold inlets.

6. Solve for the pressure distribution by solving the finite element equation, equation (18).

7. Compute the pressure sensitivity distribution, $\mathbf{S_P}$, from equation (29).

8. Note that if fill time sensitivity is to be calculated, equation (35) must be included in the computational procedure. In order for this to occur, the flow rate at the inlets must be calculated for each time step.

9. Compute the flow rate at the inlet nodes with equation (16). The pressure distribution must be known before completing this step.

10. Compute the flow rate sensitivities for all inlet nodes using equation (29).

11. Compute and sum the fill time sensitivity contribution from the current time step, equation (35).

12. Save the results and continue on to the next time step until the filling is completed.

6. Analytical Fill Time Sensitivity Solution

An analytical solution is used to verify the finite element developments. For a circular disk with a hole being filled from the center with constant pressure, the fill time solution is given as (see Figure 1(a)):

$$t = \frac{\mu}{k} \frac{\Phi}{P_0} \left[\frac{R^2}{2} \ln \left(\frac{R}{R_0} \right) - \frac{R^2}{4} + \frac{R_0^2}{4} \right] \tag{36}$$

where

$$\mu = \text{viscosity}$$

$$\Phi = \text{porosity}$$

$$k = \text{permeability}$$

$$P_0 = \text{Inlet Pressure}$$

$$R_0 = \text{Inner Radius}$$

$$R = \text{Outer Radius}$$

$$t = \text{Time to fill from } R_0 \text{ to } R$$

The other possible inlet condition with an analytical solution is for the case of constant flow rate. This analytical solution is given as

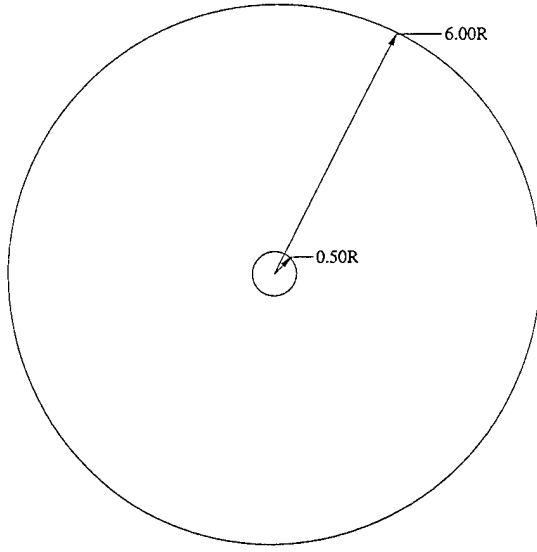$$R(t) = \left[\frac{Qt}{\pi \Phi H} + R_0^2 \right]^{\frac{1}{2}} \tag{37}$$

where the terms are the same as before, except that
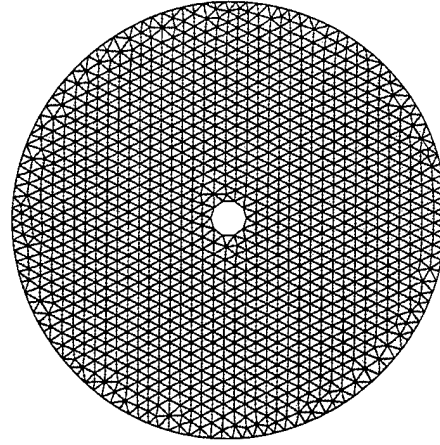
$$H = \text{Mold Thickness}$$

$$Q = \text{Inlet Flow Rate}$$

The inlet pressure for the constant flow rate boundary condition can also be computed analytically, as

$$P_0 = \frac{\mu Q}{2\pi k H} \ln \left(\frac{R(t)}{R_0} \right) \tag{38}$$

(a) 6.00-in. R Disk with 0.50-in. R hole for isothermal RTM analytical and FEM comparison

(b) Finite element mesh for 6.00-in. R Disk with 0.50-in. R hole

Figure 1: Geometry and finite element mesh used for analytical/numerical comparison.

where

$$R(t) = \text{The radius filled to at a specific time}$$

The analytical solution is described here because it can be useful in verifying the isothermal filling solution to ensure that the finite element developments are accurate. An analytical solution to the sensitivity analysis can also be computed and used to validate the subsequent CSE developments for isothermal filling. In order to obtain the analytical solution for the sensitivity analysis, the partial derivative of the fill time solution for constant inlet pressure, equation (36), with respect to the sensitivity parameter p, yields

$$
\begin{aligned}
\frac{\partial t}{\partial p} = &\left(\frac{\partial \mu}{\partial p} \left(\frac{\Phi}{k P_0} \right) + \left(-\frac{1}{k^2} \right) \frac{\partial k}{\partial p} \left(\frac{\mu \Phi}{P_0} \right) + \left(-\frac{1}{P_0{}^2} \right) \frac{\partial P_0}{\partial p} \left(\frac{\mu \Phi}{k} \right) \right) \\
&\left[\frac{R^2}{2} \ln \left(\frac{R}{R_0} \right) - \frac{R^2}{4} + \frac{R_0{}^2}{4} \right]
\end{aligned}
$$

(39)

The analytical sensitivity solution for a constant flow rate injection can also be derived from an analytical filling solution. Taking the partial derivative of equation (37) with respect to the sensitivity parameter p, yields

$$\frac{\partial t}{\partial p} = -\frac{1}{Q^2}\frac{\partial Q}{\partial p}\left[\left(R(t)^2 - R_0{}^2\right)\pi\Phi H\right] \tag{40}$$

Note that the analytical fill time solution has been derived for a circular disk with a hole at the center, as shown in Figure 1(a). This model, along with the results obtained from it, are presented as verification for the fill time sensitivity derivation.

6.1 Verification of Isothermal RTM Sensitivity Equations - Analytical and Numerical Results

The verification of the isothermal sensitivity equations is performed by comparing the results from the analytical solution, equation (39), with the results obtained from the finite element RTM developments. The model for the collected data is shown in Figure 1(b). The default variable values for the results presented in Figures 2(a) and 2(b) are the following

$$R_0 = 0.5in$$

$$R = 6.0in$$
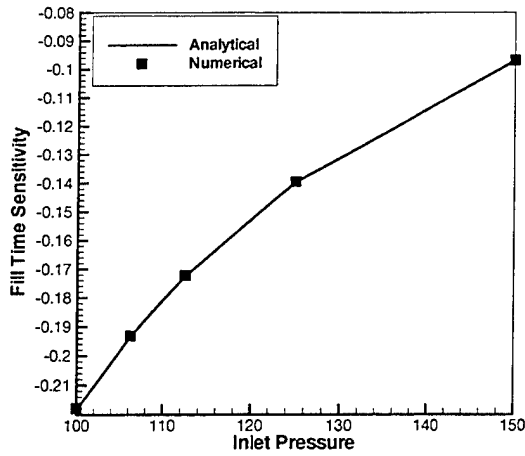
$$\mu = 7.25 \cdot 10^{-7}\frac{lbf - s}{in^2}$$

$$k = 3.565 \cdot 10^{-9}in^2$$
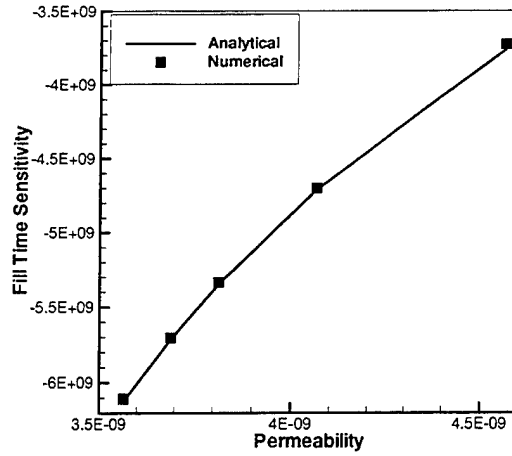
$$\Phi = 0.3$$

$$vof = 1.0 - \Phi = 0.7$$

$$P_0 = 100.0psi$$

The comparative numerical and analytical results for inlet pressure and permeability sensitivities are shown in Figures 2(a) and 2(b), respectively. The agreement of the results

(a) Comparison of analytical and numerical results for filling of the disk model for sensitivity parameter of inlet pressure - isothermal consideration

(b) Comparison of analytical and numerical results for filling of the disk model for sensitivity parameter of permeability - isothermal consideration

Figure 2: Plot of verification results - isothermal consideration.

is excellent and clearly verifies the present developments for sensitivity parameters of inlet pressure and permeability.

7. Example Sensitivity Problem Description

Illustrative fill time sensitivity results are presented next for a 2-in. by 2-in. plate shown in Figures 3(a) and 3(b). The inlet location used for this model is the bottom left corner of the plate.
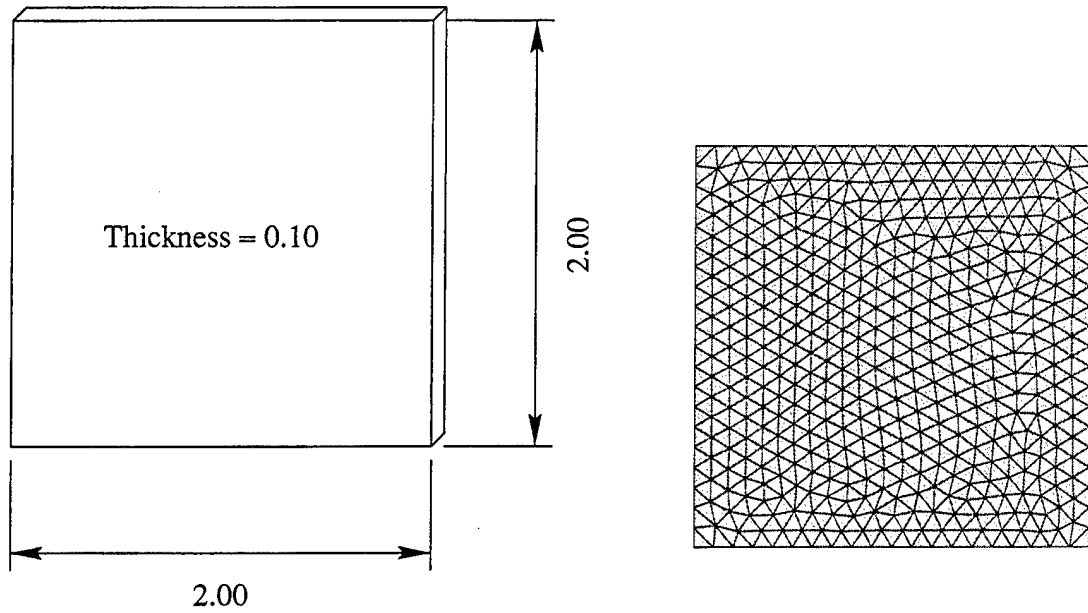
(a) 2-in. x 2-in. Plate for isothermal RTM analysis

(b) 2-in. x 2-in. Plate Mesh for isothermal RTM analysis

Figure 3: Geometry and finite element mesh for isothermal RTM analysis.

The default model values for the results presented in Figures 4–9, are the following

$$k = 3.565 \cdot 10^{-8} in^2$$

$$\mu = 2.03053 \cdot 10^{-5} \frac{lbf - s}{in^2}$$

and where applicable $\qquad$ (41)

$$P_0 = 5.0 psi$$

$$q_0 = 3.0 \cdot 10^{-4} \frac{in^3}{s}$$

Inlet pressure, pressure sensitivity, fill time, and fill time sensitivity were computed for the 2-in. by 2-in. plate. These results are shown as an illustration of the type of results one can obtain from utilizing the CSE. In Figure 4, the pressure and pressure sensitivity results are plotted for varying permeability values. The inlet pressure decreases with increasing permeability, as would be expected from analysis of the analytical solution, equation (38). The

pressure sensitivity decreases with increasing permeability, illustrating the fact that the inlet pressure is less sensitive to increases in permeability as the nominal value increases. Figure 5 plots the fill time and the fill time sensitivity versus permeability results. In Figure 5(a), the fill time decreases with increasing permeability as is expected from equation (39). Figures 6 and 7 show pressure and fill time sensitivity with respect to the resin viscosity. From the analytical solution, equation (36), it is evident that the fill time is directly proportional to the viscosity. The plots illustrate this point. Since inlet pressure is directly proportional to the resin viscosity, the slope of the line in Figure 6(a) is approximately constant and the sensitivity results should be constant, as illustrated in Figure 6(b). Figure 8 shows inlet pressure versus inlet flow rate and inlet pressure sensitivity versus inlet flow rate. The plots demonstrate the fact that inlet pressure is a linear function of inlet flow rate for this range of flow rates. Figure 9 shows the fill time and the fill time sensitivity results versus inlet pressure for the 2-in. by 2-in. plate. The trends of decreasing fill time and decreasing fill time sensitivity closely follow the analytical solutions given in equations (36) and (39), respectively.

8. Computation of Unknown Material Property

In this sample illustrative problem, the viscosity of flowing resin is assumed to be not accurately known for a particular structural part being manufactured. The numerical results are computed with the finite element discretized model and compared to the fill time measurements from the laboratory. The material properties and inlet conditions for the initial finite element model are selected as follows:

$$k = 3.565 \cdot 10^{-9} in^2$$

$$\mu = 7.25 \cdot 10^{-7} \frac{lbf - s}{in^2}$$
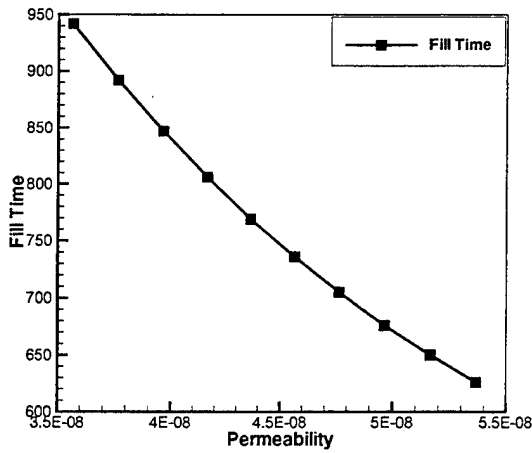
$$P_{inlet} = 100.0 psi$$
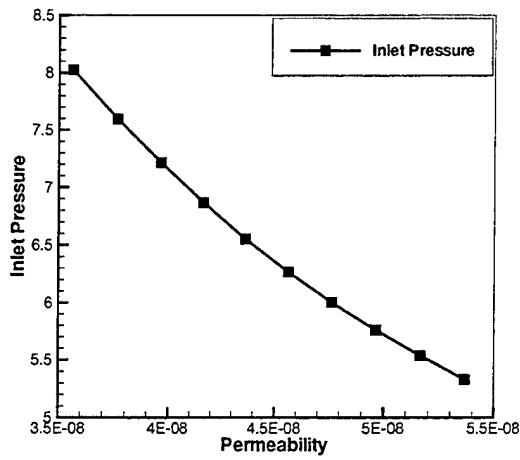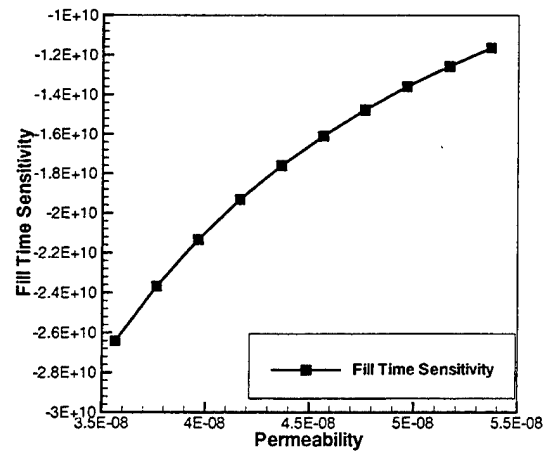
(a) Isothermal pressure vs. permeability
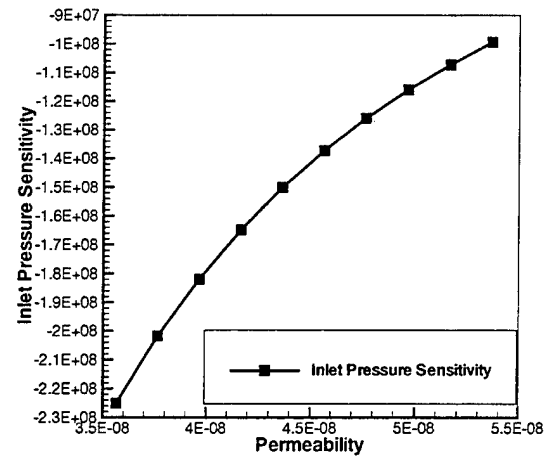
(b) Isothermal pressure sensitivity vs. permeability

Figure 4: Pressure and pressure sensitivity vs. permeability plots for isothermal RTM filling of the 2-in. by 2-in. plate.
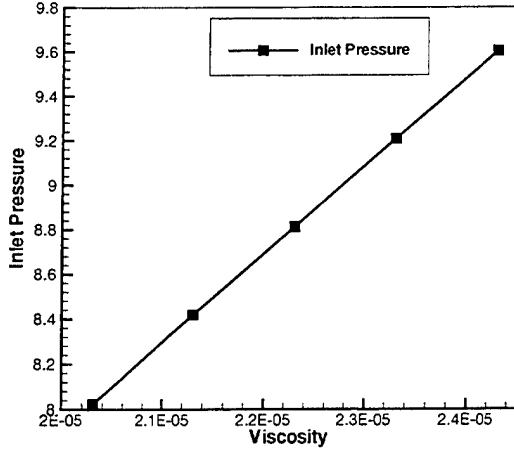

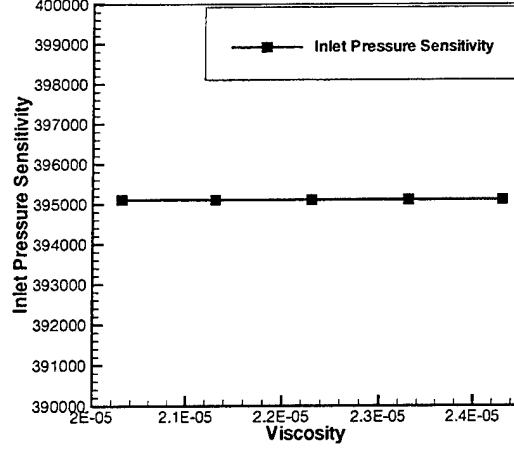(a) Isothermal fill time vs. permeability

(b) Isothermal fill time sensitivity vs. permeability

Figure 5: Fill time and fill time sensitivity vs. permeability plots.

19

(a) Isothermal inlet pressure vs. viscosity

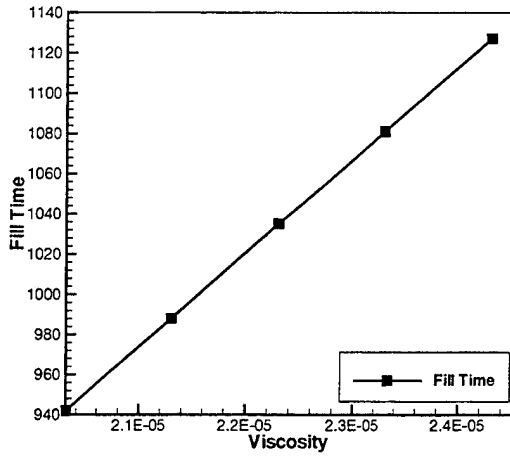(b) Isothermal inlet pressure sensitivity vs. viscosity

Figure 6: Inlet pressure and inlet pressure sensitivity vs. viscosity plots for isothermal RTM filling of the 2-in. by 2-in. plate.

The geometry and finite element model are illustrated in Figures 10(a) and 10(b), respectively.

The sensitivity results are used here to calculate the value of viscosity which was used in a laboratory experiment. During the experiment, mold filling occured in approximately 60 s. Earlier, a numerical analysis was performed using the finite element method which predicted a fill time of approximately 100 s. By combining the sensitivity results with the Newton iteration method, the unknown viscosity can be computed. The function to be solved is defined as

$$f(\mu) = t_{f_{actual}} - t_{f_{simulated}}(\mu) \tag{42}$$

In equation (42), $t_{f_{actual}}$ is the actual fill time measured in the laboratory and $t_{f_{simulated}}(\mu)$ is the currently computed fill time from the finite element model. By finding the root of
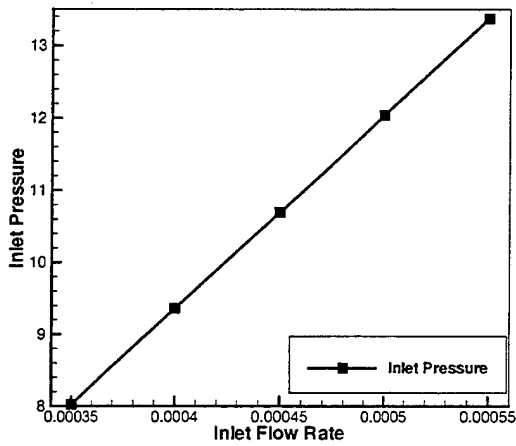
20

(a) Isothermal fill time vs. viscosity
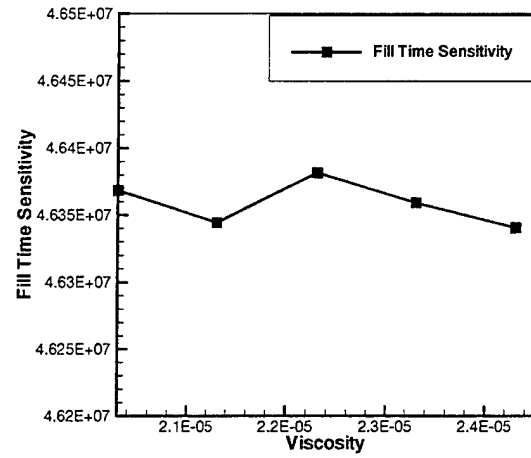
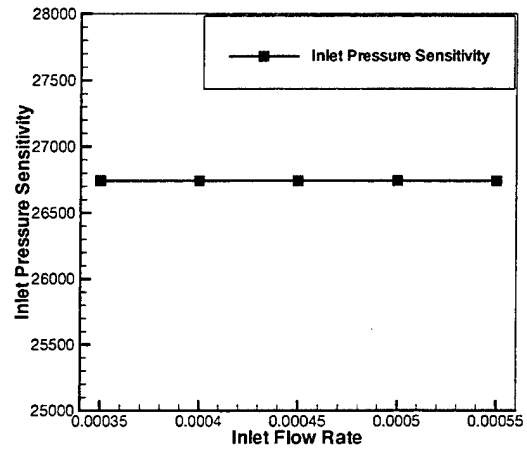(b) Isothermal fill time sensitivity vs. viscosity

Figure 7: Fill time and fill time sensitivity vs. viscosity plots for isothermal RTM filling of the 2-in. by 2-in. plate.
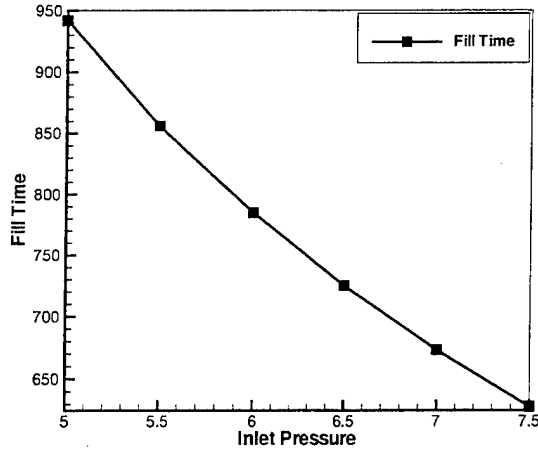

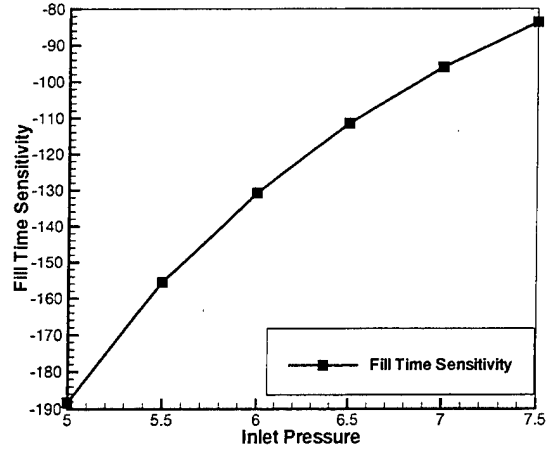(a) Isothermal pressure vs. inlet flow rate

(b) Isothermal pressure sensitivity vs. inlet flow rate

Figure 8: Pressure and pressure sensitivity vs. inlet flow rate plots for isothermal RTM filling of the 2-in. by 2-in. plate.

(a) Isothermal fill time vs. inlet pressure

(b) Isothermal fill time sensitivity vs. inlet pressure

Figure 9: Fill time and fill time sensitivity vs. inlet pressure plots for isothermal RTM filling of the 2-in. by 2-in. plate.

this equation, the viscosity from the experiment can be calculated. To calculate the root of equation (42), the Newton method is used employing

$$\mu_{n+1} = \mu_n + \frac{-f(\mu_n)}{f'(\mu_n)} \tag{43}$$

where n denotes results from the previous viscosity estimation and $n + 1$ denotes values for the current viscosity estimation. By iterating until convergence is reached, the correct viscosity values are computed, as shown graphically in Figure 11. Such techniques are of practical importance to designers and can also be applied to other analyses, including fill time optimization and inlet parameter computations.
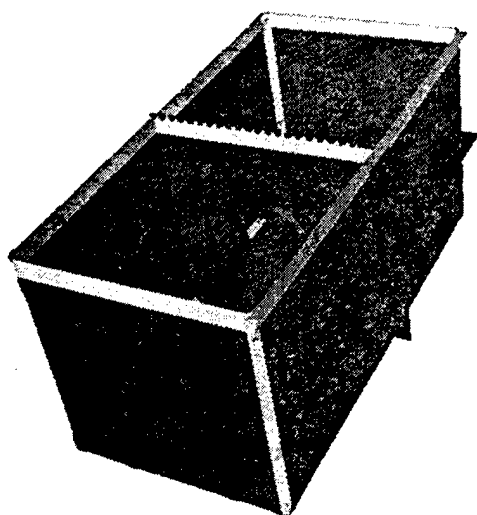
(a) Risk reduction box geometry, displaying
different material regions
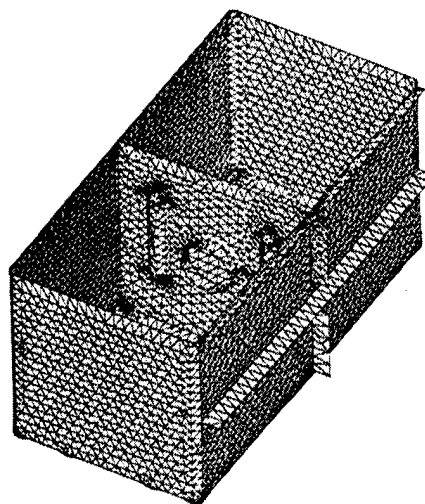
(b) Risk reduction box mesh

Figure 10: Geometry and finite element mesh used for computation of unknown material property.

9.   Concluding Remarks

Material property, boundary condition, and geometric sensitivity parameters were presented for isothermal resin transfer molding considerations. The continuous sensitivity equation was developed for the isothermal resin transfer molding process simulation studies by starting from the governing model equations and applying the finite element method. Once the CSE was formulated, a cost function, namely the fill time, was derived along with the fill time sensitivity.

Using analytical results for filling of a disk the numerical method was verified for sensitivity analysis. After verification, a simple 2-in. square plate and the corresponding sensitivity results were presented. A sample application for the sensitivity results were given in the
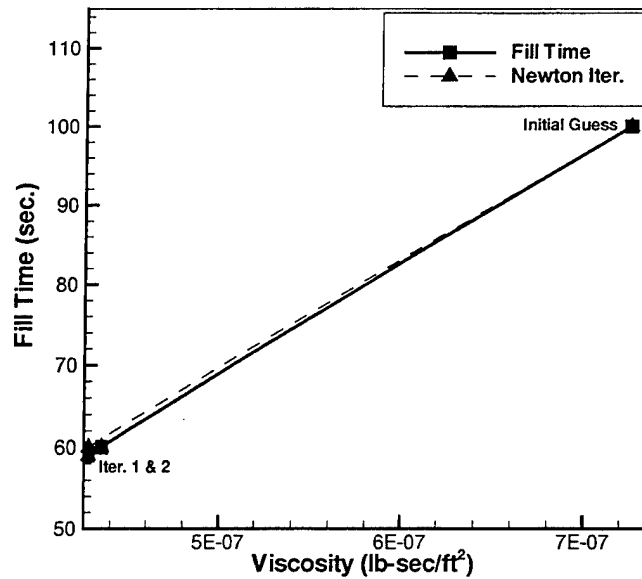
Figure 11: Fill time vs. viscosity with Newton iterations.

computation of an unknown material property of a structural part being manufactured. In the analysis the viscosity was computed for the geometry of a risk reduction box after example laboratory results did not coincide with the results from the numerical simulations. The usefulness of the present efforts as a design tool was subsequently demonstrated.

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1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	March 2002	Final, June 2000–June 2001

4. TITLE AND SUBTITLE
Process Modeling of Composites by Resin Transfer Molding: Sensitivity Analysis for Isothermal Considerations

5. FUNDING NUMBERS
1UH7AL

6. AUTHOR(S)
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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
U.S. Army Research Laboratory
ATTN: AMSRL-CI-HC
Aberdeen Proving Ground, MD 21005-5067

8. PERFORMING ORGANIZATION REPORT NUMBER
ARL-TR-2686

9. SPONSORING/MONITORING AGENCY NAMES(S) AND ADDRESS(ES)

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

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12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

The resin transfer molding (RTM) manufacturing process consists of either of two considerations; the first is the fluid flow analysis through a porous fiber preform where the location of the flow front is of fundamental importance, and the second is combined flow/heat transfer analysis. For preliminary design purposes and the case of relatively large molds, isothermal considerations seem fairly representative of the physical situation. The continuous sensitivity formulations are developed for the process modeling of composites manufactured by RTM to predict, analyze, and the optimize the manufacturing process. Attention is focused here on developments for isothermal flow simulations, and illustrative examples are presented for sensitivity analysis applications which help serve as a design tool in the process modeling stages.

14. SUBJECT TERMS
continuous sensitivity equation, resin transfer molding, finite element, RTM

15. NUMBER OF PAGES
33

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL